Measuring social polarization with ordinal and cardinal data: an application to the missing dimensions of poverty in Chile

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Abstract

We examine the measurement of polarization with categorical and ordinal data. This is particularly useful in many contexts where cardinal data are not available. The new measures we propose are axiomatically characterized. The empirical application on Chilean data show that the rankings obtained are substantially different when compared to standard polarization and inequality measures.

Key Words: polarization, axiomatic derivation, inequality.

JEL Classification Numbers: C43, D31, D63.

1. Introduction

The measurement of polarization has received increasing attention in the past years (see, among others, Foster and Wolfson, 1992, Esteban and Ray, 1994, Wolfson, 1994, Wang and Tsui, 2000, Chakravary and Majumder, 2001, Duclos, Esteban and Ray, 2004). An important reason behind this interest is the existing connection between polarization and several social, economic or political phenomena, especially those related to social tension and conflict. However, while most researchers have focused on the measurement of "income polarization" alone, only a few of them have attempted to study what might be broadly referred to as "social polarization" (see, for example, D'Ambrosi,o 2001, Zhang and Kanbur, 2001, Duclos, Esteban and Ray, 2004, and Garcia-Montalvo and Reynal-Querol, 2005) . The latter term is used when the factors that determine individuals' identity are culturally, ideologically, historically, biologically or socially driven and do not depend solely on their income levels (classical examples are ethnic, racial, nationalistic, religious or political polarization). Clearly, there is great interest in defining social polarization indices because, in many circumstances, income distribution is not the only relevant dimension that might be the cause of social conflict (see Stewart 2009, Esteban and Ray 1999, Garcia-Montalvo and Reynal-Querol 2005, Collier and Hoeffler 2001 or Easterly and Levine 1997 for some empirical or theoretical works that explore the existing links between polarization and conflict and other related issues).

Traditional income polarization measures are implicitly or explicitly based on the assumption that individuals clustered around certain income levels form a cohesive *group* that might potentially express its unrest into social action or revolt. In Esteban and Ray (1994), for instance, individuals are assumed to feel: 1) Identified with other individuals with the same income levels and 2) Alienated towards individuals with different income levels. Traditional bipolarization measures are also implicitly constructed under the assumption that the problems of a society with a declining middle class will derive from the existence of large and cohesive "poor" and "rich" classes. However, it seems clear that there are other salient characteristics (such as race, ethnicity, religious group, gender) that exert a great influence in defining individuals' sense of identity. As argued by Dasgupta and Kanbur (2007): "[…] the nominal distribution of income could give a misleading picture of tensions in society, both within and across communities. Ideologies of community solidarity may well trump those of class solidarity because of the implicit sharing of community resources brought about by community-specific public goods”.

The only polarization measure that, to our knowledge, explicitly uses only the distribution of groups on ethnic or religious lines is the Reynal-Querol index *RQ*. However, this index is only defined on the basis of the population-weight that these groups represent, but there is no information about their privileged/unprivileged status in terms of income distribution. It would be interesting to define a social polarization index that combined the intuition of both approaches: On the one hand define groups on the basis of salient social characteristics (like race, ethnicity, and so on) and on the other hand taking into account the extent to which these groups are clustered in certain regions of an attribute’s distribution. This is one of the main purposes of this paper.

Another practical problem encountered in practice is to define the notion of polarization on the basis of categorical or ordinal data. A recent contribution by Apouey (2007) proposes a solution in the context of ordinal data, applying the results to the assessment of polarization in the distribution of self-assessed health data (SAH). The polarization measure proposed in that paper is an extension of classical income bipolarization measures to the context of ordinal data. Again, this contribution is implicitly based on the assumption that the individuals in the same area of the (health) distribution form a cohesive group who might eventually express its unrest through social action or protest. To illustrate the limitations of this point of view, we will show the following hypothetical example. Assume that the population is divided in two racial groups (to simplify, Blacks and Whites) and that there are five self-reported health status: Very Poor (VP), Poor (P), Fair (F), Good (G) and Very Good (VG). Consider the following couple of self-assessed health distributions.

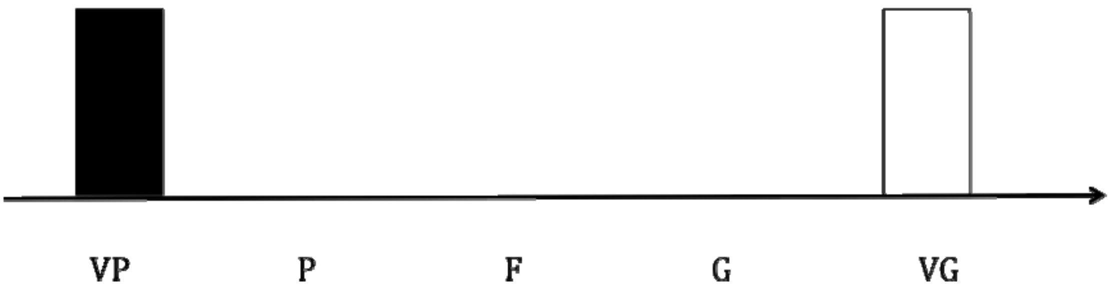


Figure 1. A hypothetical distribution of self-assessed health in two different groups.

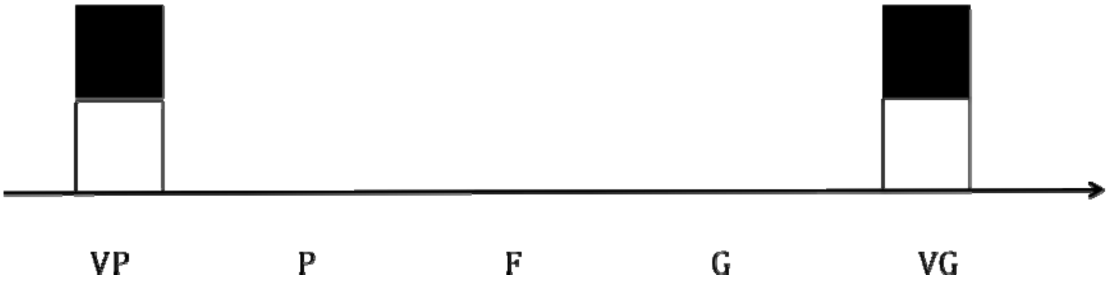


Figure 2. A hypothetical distribution of self-assessed health in two different groups.

According to the *RQ* index, most classical “income polarization indices” and Apouey’s measure, both scenarios show the same level of polarization. The first one (*RQ*) because it only takes into account the proportion of Blacks and Whites in the population disregarding the health distribution whereas the other measures *do* take into account the health distribution but not the existence of identity groups (Blacks and Whites). However, it seems intuitively clear that the scenario shown in Figure 1 (where all Blacks are underprivileged and all Whites are privileged) has higher polarization levels than the one shown in Figure 2 (where neither Blacks nor Whites are privileged) *if* it is the case that being Black or White has a strong influence in defining individuals’ identity.

In some cases one might also be interested in defining the notion of polarization in the context of categorical/nominal data. To our knowledge, such a measure has not been defined yet. However, it does not seem difficult to imagine circumstances in which the specific distribution of social groups in the different categories of a nominal variable can make a difference when measuring the levels of social polarization. Consider, as before, a population split in two racial groups, “Blacks” and “Whites” with population shares 1/2, 1/2 and a categorical variable, like place of residence or employment category. The *RQ* index will give the highest possible level of polarization (equal to 1) irrespective of the distribution of the two groups in the different categories. However, we contend that the polarization measure should be sensitive to the fact that in some cases, both racial groups can be equally represented in the different categories whereas in other cases, both groups can be completely segregated. One might intuitively argue that in the latter case, the level of social polarization should be higher than in the former.

Hence, one of the main contributions of the paper is to define new polarization measures taking into account the intuitions of classical income and social polarization measures by exploring the extent to which different social groups defined on the basis of salient characteristics are clustered in certain "privileged or underprivileged regions" of an attribute's distribution that can be measured in a categorical or an ordinal scale. The measures presented here will be axiomatically characterized. This characterization will give a normative basis that can be helpful to decide about the appropriateness of the proposed measures. Another interesting feature of the polarization measures proposed in this paper is that it is possible to decompose their values and know the specific contribution of each social group to the existing level of social polarization. Hence, it might be possible to identify the groups that contribute the most (least) to the existing levels of social tension and that could more (less) probably give rise to social activism or protest.

In order to define the new social polarization measures we will use the Identification-Alienation (IA) approach proposed by Esteban and Ray (1994). The population is assumed to be split in different exogenously given groups. Individuals will be assumed to feel identified with the members of their own social group and alienated towards the others. In the context of categorical data, alienation between social groups will be defined as a certain function of the overlap coefficient θ. In the context of continuous distributions, this coefficient is defined as

$$\theta = \int_{-\infty}^{\infty} \min\{f(x), g(x)\} dx$$

where *f* and *g* are density functions. This measure can be adapted to the context of categorical data in a straightforward way. In the context of ordinal data, alienation between groups '*i*' and '*j*' will be defined as a certain function of the following coefficient:

$$A_{ij} = \frac{\sum_{s=1}^{N_i} \sum_{t=1}^{N_j} \delta_{st}}{N_i N_j}$$

where N_i, N_j are the sizes of groups *i*, *j*, and δ_{st} is 1 if individual '*s*' from group *i* is ranked below individual '*t*' from group *j* and 0 otherwise. Recall that this is an asymmetric function ($A_{ij} \neq A_{ji}$), thus indicating that alienation felt from underprivileged groups towards more privileged ones will not be exactly reciprocated (this is in contrast with traditional income polarization measures, where alienation is symmetric).

After deriving these new measures, we will provide an empirical illustration using data from the first nationally representative survey carried out in Chile during 2008/2009 from the "Missing Dimensions of Poverty" project that has been created under the supervision of the Oxford Poverty & Human Development Initiative (OPHI). Interestingly, most of the variables in the Missing Dimensions identified by OPHI researchers are of ordinal or categorical nature. In this context, the polarization measures proposed in this paper will be particularly useful. Moreover, it will be interesting to compare polarization levels with respect to traditional income polarization levels. It will be then possible to identify which are the dimensions that divide the Chilean society more deeply.

The literature on the measurement of polarization has always been closely related and inspired by the measurement of inequality. In some papers, the authors have even argued that there is no substantial difference between inequality and polarization measures (Zhang and Kanbur 2001) while other authors have found substantial differences between both measures (Apouey 2007). Hence, it will be interesting to contribute further to this debate by exploring the extent to which the inequality and polarization measures give consistent/similar results or not. In order to measure inequality levels with ordinal data we will transform the underlying ordinal variable into a cardinal one by means of ordered logit models. Recent examples of this technique can be found in Apouey (2007) and Kirschmann & Schneider (2007).

2. New polarization measures for cardinal and ordinal data.

In this section we will present some new polarization measures that can be used in the context of cardinal and ordinal data. For that purpose, we will need to introduce some notation.

We assume that our population has N individuals and that this population is partitioned in k exogenously given groups $\boldsymbol{G}:=\{G_1,\dots,G_k\}$. We require the partition to be faithful to individual's identity feelings, that is: we assume that the members of any group feel idenified with their peers within their group. Typically, the lines along which these groups might be defined are ethnic or religious, but we leave its character underspecified to make room for other potentially useful partitions. The population shares of these groups will be denoted by $\pi_1,\dots,\pi_k$ respectively and their absolute size by $N_1,\dots,N_k$.

Both in the contexts of categorical and ordinal data, we assume that the individuals of the population belong to C different categories (in the context of ordinal data, these categories are ordered according to a certain criterion). In order to describe the distribution of the k groups in the C categories we define $p_{G_i,c}$ as the share of group G_i that belongs to category c.

By definition,

$$\sum_{c=1}^{C} p_{G_i,c} = 1$$

For each category c, there are M_c individuals. Moreover, one has that

$$M_c = \sum_{i=1}^{k} N_i p_{G_i,c} \quad \text{and} \quad \sum_{c=1}^{C} M_c = N.$$

In this paper we will use a certain version of the Identification-Alienation approach presented in ER and later used in DER among others. In this framework, individuals are assumed to feel identified with the members of their own group and alienated towards the members of the other groups. Our underlying assumption is that each group constitutes a homogeneous body whose members cannot be distinguished from each other when one tries to measure social tension. On the one hand, the identification component for each member of the group depends on the size of the group to which s/he belongs. On the other hand, and given the fact that the members within a group are indistinguishable between them, the alienation component between any individual from group G_i and any any individual from group G_j will be assumed to be the same, so we can talk about alienation between groups rather than alienation between individuals.

Now, the measurement of alienation between groups will depend on the context we are working with. If we are working with categorical data, it will be assumed that alienation between members of different groups will depend on the extent to which the respective group representation in the different categories overlap. The greater the degree of overlap, the greater the similarity between the groups, so the smaller the alienation between them. At the other extreme, the lack of overlap between two groups can be interpreted as greater difference and alienation between them. This kind of motivation is also found in segregation indices, which have typically been defined to compare distributions between women and men or between black and white (see, for instance, Charles and Grusky (1995, 1998)). When one group is concentrated in certain categories where the other is absent and vice versa, the behavior of the groups is very different, which should translate into a higher level of animosity/alienation between them.

The use of the overlap measure in the measurement of polarization is not new. For instance, Anderson et al (2010) have used it in the case of continuous income distributions. In this context, we will define the measure of overlap between groups G_i,G_j as

$$\theta_{ij} = \sum_{c=1}^{C} \min\{p_{G_i,c}, p_{G_j,c}\}$$

By definition, the overlap coefficient lies between 0 (disjoint groups) and 1 (perfectly overlapping groups). The most natural way of defining alienation with the overlap coefficient is to define $1-\theta_{ij}$, so that it takes the value of 0 when the groups overlap completely and the value of 1 when the groups are completely disjoint.

When one is working with ordinal information one could follow the same approach as before, that is: use the overlap coefficient as a measure of lack of alienation between groups. This would result in a symmetric measure, where the alienation from G_i to G_j would be the same as from G_j to G_i (because $\theta_{ij} = \theta_{ji}$). However, one might well be interested in introducing asymmetric alienation functions. It has been argued elsewhere that the feeling of alienation between groups should not be necessarily reciprocated. Consider, for instance, a comparison between a poor and a rich individiual: while the poor has good reason to feel animosity towards the rich, the opposite feeling might not be the same. In this context, we are assuming that the ordinal variable one is working with represents some *desirable* attribute (say health, income or education), so that the feeling of alienation will be felt from poorer to richer individuals but not the other way round (the symmetric case has already been covered by the overlap coefficient). Hence, alienation between groups '*i*' and '*j*' will be defined as a certain function of the following coefficient:

$$A_{ij} = \frac{\sum_{s=1}^{N_i} \sum_{t=1}^{N_j} \delta_{st}}{N_i N_j}$$

where δ_{st} is 1 if individual '*s*' from group *i* is ranked below individual '*t*' from group *j* and 0 otherwise. Recall that this is an asymmetric function ($A_{ij} \neq A_{ji}$), thus indicating that alienation felt from underprivileged groups towards more privileged ones will not be exactly reciprocated (this is in contrast with traditional income polarization measures, where alienation is symmetric). The value of A_{ij} measures the extent to which group G_i is underprivileged with respect to group G_j. When A_{ij}=1, all the members of group G_i are ranked below any member of group G_j with respect to the ordinal attribute we are taking into account. This would be the case of maximal alienation. At the other extreme, A_{ij}=0 when no member of group G_i is ranked below any member of group G_j, which is the case of minimal alienation.

According to the IA approach, the effective antagonism felt between two individuals can be measured with a function *T(i,a)*, where it is assumed that *T* is continuous, increasing in its second argument and *T(i,0)=T(0,a)=0*. Finally, total polarization is postulated to be proportional to the sum of all efective antagonisms, that is:

$$P(G) \equiv \sum_{s=1}^{k}\sum_{t=1}^{k} N_s N_t T(s,a) \qquad (1)$$

This is a very general expression that can be adapted to the different contexts we want to work with. In the context of categorical data or ordinal data with symmetric alienation, and under the aforementioned assumptions, the last expression can be rewritten as

$$P_S(G) \equiv \sum_{s=1}^{k}\sum_{t=1}^{k} N_s N_t T(N_s, 1-\theta_{st}) \qquad (2)$$

whereas in the context of ordinal data with asymmetric alienation one has that

$$P_A(G) \equiv \sum_{s=1}^{k}\sum_{t=1}^{k} N_s N_t T(N_s, A_{st}) \qquad (3)$$

2.1. Axioms and characterization results.

At this moment, we introduce the axioms that will be used to characterize and give more specific functional forms to our polarization measures. Interestingly, in some cases our axioms can be presented in a general way for the categorical, ordinal-symmetric and ordinal-asymmetric cases.

Axiom 1. Consider a two group society in time t_0 with π_1 being greater than π_2, two categories *c*=1,2 (in the ordinal case we consider that the second category represents a higher achievement level than the first) and $p_{G_1,1}=1, p_{G_1,2}=0, p_{G_2,1}=d, p_{G_2,2}=1-d,$ for some 0<*d*<1. Assume now that after some time t_1, the second group splits into two equally sized groups $\tilde{G}_2, \tilde{G}_3$, with $p_{\tilde{G}_2,1}=d-\varepsilon, p_{\tilde{G}_2,2}=1-d+\varepsilon, p_{\tilde{G}_3,1}=d+\varepsilon, p_{\tilde{G}_3,2}=1-d-\varepsilon,$ for some arbitrarily small $\varepsilon < d$. After such split, polarization should not increase.

The intuition behind this axiom is the following. Before the distributional change, there is a big group in only one category (the "poor category" in the ordinal case) and a smaller group that is distributed between the first category (with a small presence there) and the second one (the "rich category" in the ordinal case). After some time, the small group G_2

breaks down in two equally sized groups $\tilde{G}_2, \tilde{G}_3$ in such a way that the average animosity from the large group G_1 towards the new subgroups is the same as the original animosity with respect to G_2. Given the fact that the average animosity is kept the same and that the opposition that G_2 might have created against the bigger group G_1 has been dilluted by its division in two smaller subgroups, one might expect that polarization should decrease. Figure 1 illustrates this axiom.

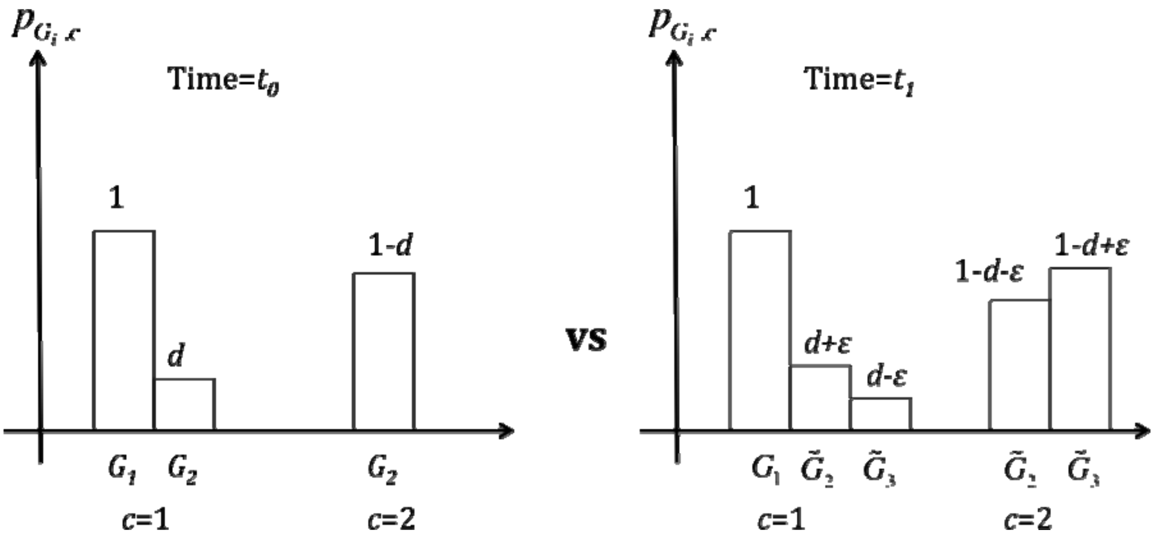


Figure 1. Illustration for Axiom 1.

Axiom 2. Consider a three-group society in time t_0 with π_1 being greater than π_2, π_3, two categories c=1,2 and $p_{G_1,1}=1, p_{G_1,2}=0, p_{G_2,1}=d, p_{G_2,2}=1-d, p_{G_3,1}=0, p_{G_3,2}=1$, with d<0.5. Assume now that, after some time t_1, the distribution of groups in the two categories is $p_{G_1,1}=1, p_{G_1,2}=0, p_{G_2,1}=d-\varepsilon, p_{G_2,2}=1-d+\varepsilon, p_{G_3,1}=0, p_{G_3,2}=1$, for some arbitrarily small $\varepsilon<d$. After such a change, polarization should not decrease.

Figure 2 illustrates this axiom. Initially, there is a large group G_1 and two smaller groups, G_2, G_3, in such a way that the animosity from G_1 towards G_2 is greater than the animosity from G_2 towards G_3. After some time, the animosity from G_1 towards G_2 is even larger and the animosity from G_2 towards G_3 is even smaller. In such a case one would expect polarization to increase because the smaller groups G_2, G_3 can be seen as a more cohesive opposition to the larger group G_1.

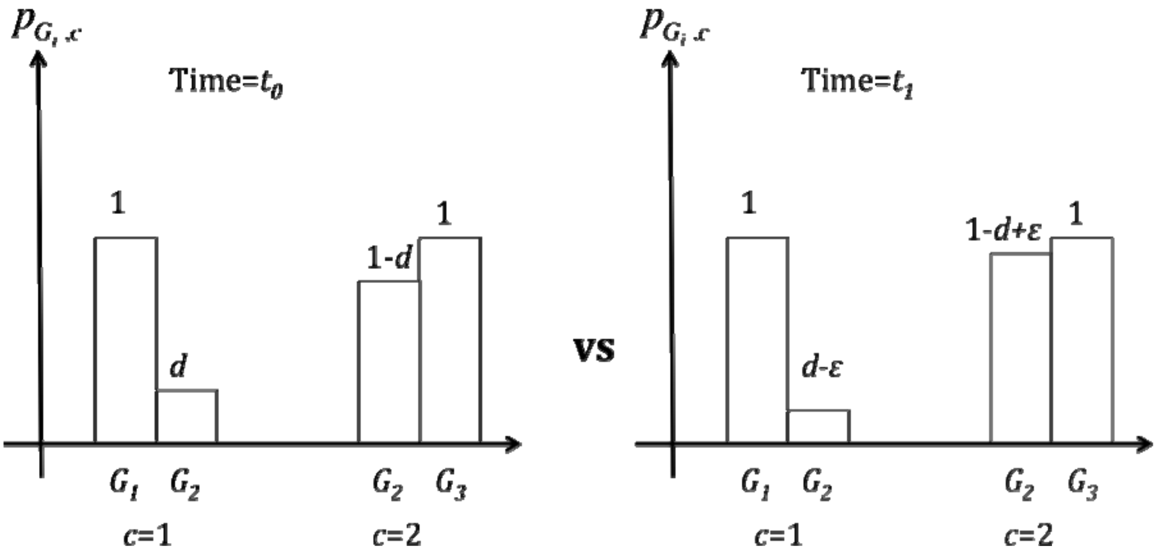


Figure 2. Illustration for axiom 2.

Axiom 3. If $P(\boldsymbol{G}_1) \geq P(\boldsymbol{G}_2)$ and $q>0$, then $P(q\boldsymbol{G}_1) \geq P(q\boldsymbol{G}_2)$, where $q\boldsymbol{G}_1$, $q\boldsymbol{G}_2$ represent population scalings of $\boldsymbol{G}_1$,$\boldsymbol{G}_2$ respectively.

This is a common invariance axiom in the literature of poverty, inequality and polarization measurement. It just states that if populations are scaled up or down, the comparisons between societies should remain the same. In particular, this allows to make meaningful comparisons between societies of different absolute sizes.

The following couple of axioms have to be stated in its symmetric (S) and asymmetric (A) alienation versions (that is, when alienation is measured by $1-\theta_{ij}$ or A_{ij} respectively), even if their underlying meaning is exactly the same.

Axiom 4S. Assume symmetric alienation. For any population of fixed mass *N* and an arbitrarily large number of categories *c*=1,2,… consider a distribution where $p_{G_i,i}=1$ for all G_i□{G_1,…,G_k} (so that $p_{G_i,j}=0$ for all $j \neq i$). Then, an increase in the values of *k* will not increase polarization.

Axiom 4A. Assume asymmetric alienation. For any population of fixed mass *N* and any distribution where $p_{G_i,c}=p_{G_j,c}$ for any *c*, all G_i,G_j□{G_1,…,G_k} and $p_{G_i,c}>0$ for at least two different categories, an increase in the value of *k* will not increase polarization.

These axioms capture the widespread idea that, other things being equal, the larger the number of groups, the lower the corresponding polarization. The intuition behind these axioms is the following: as the different groups become smaller (since the size of the population is fixed and the number of groups increases) and the alienation between these groups is kept constant, their members have less power to effectively voice their unrest, thus decreasing the level of social tension. Some authors have used this idea or very similar ones in the study of conflict and polarization (see, for example, Esteban and Ray (1994, 1999) or Montalvo and Reynal-Querol (2005), who trace this idea from the seminal works of Horowitz (1985)). It is important to recall that this axiom would not make sense if our purpose were to measure bipolarization, as is the case, for example, of the Wolfson Index (see Wolfson (1994)).

Again, the following axiom has to be stated in the symmetric and asymmetric alienation versions.

Axiom 5S. Assume symmetric alienation. Consider a three-group distribution $\{G_1,G_2,G_3\}$ with respective sizes $N_1>N_2=N_3>0$ and $p_{G_i,i}=1$ for $G_i \in \{G_1,G_2,G_3\}$. Then a population mass transfer from G_1 to G_2 and G_3 by the same amount without altering the rank of the sizes of the groups will not decrease polarization.

Axiom 5A. Assume asymmetric alienation. Consider a three-group distribution $\{G_1,G_2,G_3\}$ with respective sizes $N_1>N_2=N_3>0$ and $p_{G_i,c}=p_{G_j,c}$ for any c, all $G_i,G_j \in \{G_1,G_2,G_3\}$ and $p_{G_i,c}>0$ for at least two different categories. Then a population mass transfer from G_1 to G_2 and G_3 by the same amount without altering the rank of the sizes of the groups will not decrease polarization.

The underlying intuition behind these axioms is the same. In the process of transferring population mass from the big group to the smaller ones, the groups become gradually similar, thus equating their relative forces and increasing the tension between them. It seems reasonable to say that, other things being equal, a distribution with three equally populated and equidistant groups is more likely to exhibit higher levels of social tension than another one in which one of the groups happens to be much more populated than the other two.

Axiom 6. For any grouping of the population and any distribution, $0 \le P_S(\boldsymbol{G}) \le 1$ and $0 \le P_A(\boldsymbol{G}) \le 1$.

This is just a classical normalization assumption bounding the value of our polarization indices between zero and one.

With these axioms it is now possible to present our characterization results.

Theorem 1. A social polarization measure as defined in equation (2) satisfies axioms 1,2,3,4S, 5S and 6 if and only if it is equal to

$$P_S(G) = 4\sum_{s=1}^{k}\sum_{t=1}^{k}\pi_s^{1+\alpha}\pi_t(1-\theta_{st}) \qquad (4)$$

where $\alpha \in [\alpha^*,1]$, *with* $\alpha^* = \dfrac{2-\log_2 3}{\log_2 3 - 1} \approx 0.71.$

Proof: See the Appendix

Theorem 2. A social polarization measure as defined in equation (3) satisfies axioms 1,2,3,4A, 5A and 6 if and only if it is equal to

$$P_A(G) = \frac{27}{4}\sum_{s=1}^{k}\sum_{t=1}^{k}\pi_s^{1+\alpha}\pi_t A_{st} \qquad (5)$$

where $\alpha \in [\alpha^*,1]$, *with* $\alpha^* = \dfrac{2-\log_2 3}{\log_2 3 - 1} \approx 0.71.$

Proof: See the Appendix.

These theorems characterize axiomatically our new polarization measures. The constants by which these indices are multiplied normalize their values between zero and one (this is discussed in more detail below). It is important to recall that the value of parameter alpha can be seen as an index of polarization sensitivity: if alpha were allowed to take the value of zero (which is *not* the case for the range of admissible values obtained in Theorems 1 and 2), the indices would be equivalent to a fractionalization index[1]. Observe that when $\alpha = 1$ and when there is no overlap between the different groups (so $\theta_{ij} = 0$) $P_S(G)$ reduces to the well-known *RQ* index.

It is important to identify the distributions that maximize and minimize the polarization levels of our new measures. This way, one is able to recongnize the two most distant extremes of the concept of social polarization. In this respect, we have the following propositions.

Proposition 1. For $P_S(G)$, polarization is minimized ($P_S(G)$=0) when there is only one group and it is maximized ($P_S(G)$=1) when there are two equal-sized and non-overlapping groups.

Proof: See the Appendix.

Proposition 2. For $P_A(G)$, polarization is minimized ($P_A(G)$=0) when there is only one group and it is maximized ($P_A(G)$=1) when there are two non-overlapping groups G_1,G_2 with A_{12}=1, A_{21}=0 and $\pi_1 = 2/3, \pi_2 = 1/3$.

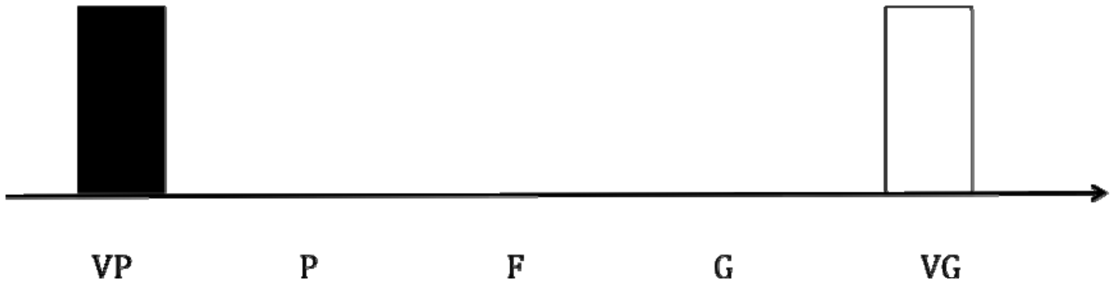
Proof: See the Appendix.

Having defined our new polarization measures $P_S(G)$ and $P_A(G)$ (or P_S, P_A from now onwards) it will also be interesting to compare their values with other indices (like the *RQ* index or Apouey's index P_2) for the motivating examples presented in the introduction.

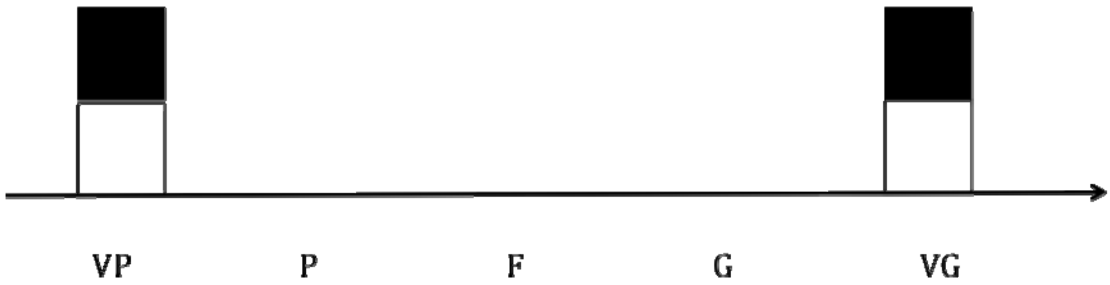
[1] Recall that a fractionalization index in this context would be written like this:

$$FRAC = \sum_{s=1}^{k} \sum_{t=1, t \neq s}^{k} \pi_s \pi_t$$

Scenario 1:



Scenario 2



It is straightforward to check that both Apouey's index (P_2) and the RQ index show the same level of polarization in both cases: it is equal to the maximal value of 1. This lack of sensitivity to such different scenarios is somewhat disturbing. However, P_S =1 , P_A =0.84375 in scenario 1 and P_S =0.5 , P_A =0.421875 in scenario 2, that is: the levels of polarization for both P_S and P_A are halved when passing from the first to the second scenario . The direction of these changes seems to be in line with our intuitions on how a polarization index should behave under such circumstances.

3. Empirical illustration.

In this section we show an empirical application of our new polarization indices. Among other things, we want to compare the values of the new indices with other polarization and inequality measures. For that purpose, we will be using data from the first nationally representative survey carried out in Chile during 2008/2009 from the "Missing Dimensions of Poverty" project that has been created under the supervision of the Oxford Poverty & Human

Development Initiative (OPHI).

For our application we will work with individuals' self-assessed health (SAH), an important subjective variable widely used in the analysis of health measures (see, for instance, Apouey (2007)). This is an ordinal variable measured in a 1 to 5 scale, 1 denoting the best possible health ('Excellent') and 5 the worst ('Very Poor'). Many longitudinal studies highlight that individuals' own appraisal of health is a very good predictor of future mortality and morbidity. Moreover, the correlation between SAH and mortality remains important even after controlling for other health and socio-economic variables. This variable will be used to compute Apouey's polarization index (P_2) within the 12 Chilean regions in which the country is divided[2] (see Table 1).

In an important paper, Montalvo and Reynal-Querol (2005) suggest that the polarization levels according to the ethnic divide in different world countries is a good predictor of the occurrence of conflict. On the other hand, the recent detailed report by the Chilean *Fundación para la Superación de la Pobreza* "Social Thresholds for Chile: Towards Future Social Policy" highlights the underprivileged situation of those individuals with partly indigenous descent (see chapter 8). In this respect, it will be interesting to explore the polarization levels that are observed when one takes into acount the ethnicity of individuals. This variable has been included in the OPHI questionnaire by asking 'In Chile the law recognises the existence of indigenous peoples; do you belong or descend from any?'. The ethnicity variable will be used to compute the Reynal-Querol (RQ) index within the 12 Chilean regions in which the country is divided (see Table 1).

Using jointly the ordinal SAH *and* the categorical ethnicity variable we will compute our new polarization indices P_S, P_A for the different Chilean regions and compare their values with P_2 and RQ. This way, we will be able to compare the rankings of the regions according to the values of the different polarization indices (see Table 1).

Another of the purposes of this empirical section is to contribute to the debate related to the redundancy (or lack thereof) of polarization vis-à-vis inequality measures. Basically, we want to test whether the region ranking that arises from classical inequality measures is consistent/similar with the ranking that arises from the values of polarization measures. In

[2] These regions are: I Tarapacá, II Antofagasta, III Atacama, IV Coquimbo, V Valparaíso, VI O'Higgins Rancagua, VII Maule Talca, VIII Biobío Concepción, IX Araucanía Temuco, X Los Lagos Puerto Montt, XI Aysén Coyhaique, RM Metropolitana. The region XII Magallanes Punta Arenas has not been included in the sample because of its small size.

order to compute the values of an inequality index using ordinal information we will transform the underlying ordinal variable into a cardinal one by means of ordered logit models. After that we compute the Gini index (*G*) and the classical "continuous income polarization index" P_f proposed by Duclos, Esteban and Ray (2004). In order to construct the cardinal health variable using an ordered logit model we will estimate SAH as a function of different health problems. The variables measuring these health problems are binary, they indicate the presence of one of the following issues: High blood pressure, Diabetes mellitus (type A or B), Acute respiratory infection, Integral oral health, Heart attack, Chronic terminal kidney insufficiency, Pacemaker or Other illnesses. If we denote by y^* the latent variable underlying SAH and *v* is one of the SAH values, we assume that y^* has various threshold points k_i (with $k_i \leq k_{i+1}$) such that $SAH_i = v \quad if \quad k_{v-1} < y_i^* \leq k_v, \quad v = 1, \ldots, 5$. We estimate this model assuming that

$$y_i^* = \sum_{j=1}^{m} \beta_j X_{ji} + \varepsilon_i$$

where the X_j are the binary variables representing different health issues.

To sum up: in Table 1 we show the values of Duclos, Esteban and Ray's P_f and the Gini index (for their computation we only need to take into account the cardinalized distribution of the ordinal attribute measured with the SAH variable), the *RQ* index (only takes into account ethnic distribution), Apouey's P_2 index (only takes into account the distribution of the ordinal attribute) and our new indices P_S and P_A (taking into account ethnic distribution *and* the distribution of the ordinal attribute SAH) within each of the 12 administrative regions of Chile. Within each cell we also show the corresponding ranking of the region according to the values of the index.

Table 1. Polarization and inequality measures for 12 Chilean Regions

Region name	DER	GINI	RQ	P2	PS	PA
Tarapacá	0,370 (9)	0,647 (1)	0,876 (1)	0,249 (9)	0,131 (4)	0,458 (1)
Antofagasta	0,329 (12)	0,530 (12)	0,571 (3)	0,273 (6)	0,193 (2)	0,242 (5)
Atacama	0,392 (6)	0,589 (4)	0,525 (4)	0,271 (7)	0,197 (1)	0,118 (6)
Coquimbo	0,414 (4)	0,632 (2)	0,268 (8)	0,234 (11)	0,104 (5)	0,045 (10)
Valparaíso	0,367 (10)	0,585 (5)	0,111 (10)	0,298 (4)	0,035 (9)	0,009 (12)
O'Higgins Rancagua	0,395 (5)	0,585 (6)	0,015 (12)	0,281 (5)	0,017 (12)	0,018 (11)
Maule Talca	0,486 (1)	0,548 (10)	0,101 (11)	0,232 (12)	0,030 (10)	0,048 (9)
Biobío Concepción	0,389 (7)	0,597 (3)	0,299 (7)	0,311 (2)	0,055 (8)	0,114 (7)
Araucanía Temuco	0,428 (3)	0,535 (11)	0,736 (2)	0,268 (8)	0,082 (6)	0,355 (2)
Los Lagos Puerto Montt	0,373 (8)	0,563 (9)	0,505 (5)	0,243 (10)	0,067 (7)	0,285 (4)
Aysén Coyhaique	0,453 (2)	0,566 (8)	0,447 (6)	0,328 (1)	0,156 (3)	0,300 (3)
Metropolitana	0,354 (11)	0,577 (7)	0,173 (9)	0,301 (3)	0,022 (11)	0,095 (8)

Note: Within each cell we have the values of the corresponding index and the corresonding ranking written between parentheses. For the computation of P_S and P_A we have taken $\alpha = 1$. Authors' calculations using OPHI dataset on Chile (2008).

As we can observe in Table 1, the rankings of the 12 Chilean regions can be extremely different depending on the measure we are working with. The region where Santiago de Chile is located (Metropolitana, which is the most populous region of the country) shows relatively high polarization levels according to Apouey's P_2 measure (that only takes into account the self-reported health status distribution) but relatively low values according to Duclos, Esteban and Ray's polarization index (DER) and the P_S measure introduced in this paper (that takes into account the self-reported health status *and* the ethnic distribution). The other regions of the country show also important variations in their relative ranking positions: for instance, Tarapacá is the most unequal region according to the Gini index and the most polarized one according to the *RQ* and P_A indices but shows relatively low polarization levels according to DER and P_2.

In order to have a more precise idea of the extent to which the different rankings are similar/dissimilar to one another we will make use of a rankings mobility function introduced in D'Agostino and Dardanoni (2009) that can be thought as a rankings distance function. It is defined as follows:

$$d(R,R') = \frac{1}{(n^3 - n)/3} \sum_{i=1}^{n} (R_i - R'_i)^2$$

where $R=(R_1,\dots,R_n)$ is a ranking of the n=12 regions, where $R_i \in \{1,\dots,n\}$ denotes the ranking of region *i* and $R_i \neq R_j$ for all $i \neq j$. Two different rankings will typically be written as *R*, *R'*. This measure is normalized and goes from 0 to 1. The value of zero is obtained when the two rankings that are being compared are exactly the same and the value of one is obtained when one compares two completely opposite rankings. It is important to point out that this ranking distance function has been axiomatically characterized using some relatively simple and quite unexceptionable axioms (for more details see D'Agostino and Dardanoni (2009)).

This way, it will be possible to compare the extent to which the choice of picking one measure or another has a great difference in terms of rankings or not. At the same time, it allows to explore what are the couples of indices whose rankings are further appart. In Table

2, we show the values of the rankings distance function *d(R,R')* when it is applied to the different rankings shown in Table 1.

It is interesting to observe that in general the distances between the different rankings are relatively high, that is: the rankings of the regions that are obtained when using the different polarization and inequality measures are substantially different. Most comparisons are within the range 0.47-0.63. Interestingly, the distances between the GINI ranking and all other polarization measures rankings are relatively high, thus suggesting that polarization and inequality are not redundant concepts. The only rankings that are at a relatively low distance are those of *RQ*, P_S and P_A. The fact that P_S and P_A provide similar rankings is not very surprising because both measures are based on the same partitioning of the population (while some of the other measures are based on alternative partitionings).

Table 2. Distances between different rankings

	DER	GINI	RQ	P_2	P_S	P_A
DER	0	0.545	0.601	0.633	0.514	0.49
GINI	0.545	0	0.524	0.475	0.479	0.594
RQ	0.601	0.524	0	0.566	0.111	0.056
P_2	0.633	0.475	0.566	0	0.538	0.528
P_S	0.514	0.479	0.111	0.538	0	0.199
P_A	0.49	0.594	0.056	0.528	0.199	0

Note: Authors' calculations using the rankings information from Table 1.

4. Summary and concluding remark

In this paper we have presented new polarization indices that can be used in the context of categorical or ordinal data. This is particularly useful because in many contexts cardinal information is not available. The new measures are axiomatically characterized, thus providing a normative justification that can be used to gauge their appropriateness vis-à-vis other polarization measures presented in the literature, like the Montalvo and Reynal-Querol index (2005), the income polarization indices proposed by Duclos, Esteban and Ray (1994, 2004) or Apouey's (2007) P_2 index. An empirical application is provided using data from Chilean data: our results suggest that the rankings provided by our new polarization indices when compared to other polarization and inequality measures can be substantially different.

These results are relevant because they contribute to the existing debate on the redundancy (or lack thereof) of polarization vis-à-vis classical inequality measures.

Appendix

Proof of Theorem 1

Proof of Theorem 2

Proof of Proposition 1

Proof of Proposition 2

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